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Abstract

We study the nuclear (A) dependence of the European Muon Collaboration (EMC) effect at high values of x ($x \geq 0.6$). Our approach makes use of conventional nuclear degrees of freedom within the Relativistic Impulse Approximation. By performing a non-relativistic series expansion we demonstrate that relativistic corrections make a substantial contribution to the effect at $x \gtrsim 0.6$ and show that the ratio of neutron to proton structure functions extracted from a global fit to all nuclei is not inconsistent with values obtained from the deuteron.

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The famous high energy deep inelastic lepton scattering results obtained in 1983 at CERN by the European Muon Collaboration (EMC) [1] and at SLAC in 1984 [2] (now known as the EMC effect) showed that quark momentum distributions are modified in the nuclear medium. The scattering is well described by assuming that the scattered lepton interacts with a bound quark by exchanging a virtual quanta with four-momentum $Q^2 = q^2 - \nu^2$, where $q = |\mathbf{q}|$ and ν are the three-momentum and energy carried by the quanta, respectively. The inclusive cross section depends on the Bjorken variable $x = Q^2/(2M_N\nu)$, which is identified with the fraction of the total longitudinal momentum carried by the struck quark. In this Letter we will discuss recent progress in the interpretation of the data for large quark momentum fractions $x \gtrsim 0.6$.

Figure 1 shows the ratio $R_A(x)$ of the nuclear structure function per nucleon, $F_2^A(x)/A$, to the deuteron structure function per nucleon, $F_2^D(x)/2$. The data are for an iron nucleus, but the results are similar for all nuclei with mass number $A \geq 3$. If the nuclear medium had no effect on the quark momentum distribution, the ratio would be unity; the (up to 20%) deviations of the nuclear structure functions are direct evidence for the effect of the nuclear medium.

Throughout the years, a variety of models have been proposed to explain the EMC effect at large x (for a review see [3]). Some invoke additional (sometimes exotic) components of the nuclear wave function, while other, more conventional models describe the EMC effect in terms of nucleon binding. In this latter picture,

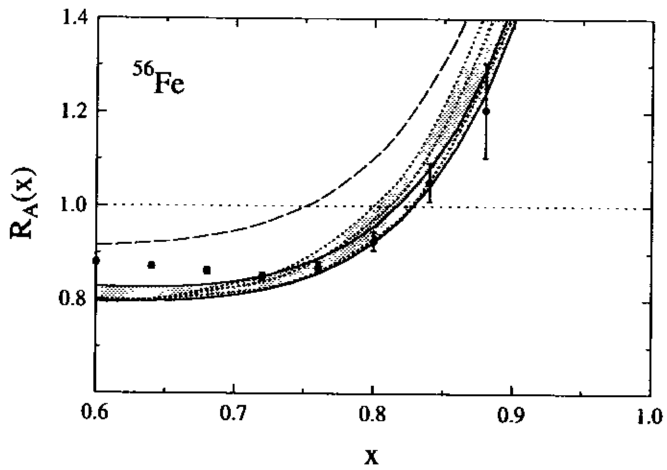

Figure 1: The EMC data for ^{56}Fe. The dashed line is the nonrelativistic calculation of CL. The shaded area is the relativistic calculation of GL, including an estimate of theoretical uncertainties.

the rise above one as $x \to 1$ is due to Fermi motion. In Ref. [4], referred to as CL in this paper, it was pointed out that a nonrelativistic calculation based on binding plus short range nucleon-nucleon (NN) correlations (generated by the NN potential) were sufficient to account for most of the effect at $x < 0.5$, but the result for large x (the long dashed line in Fig. 1) was much too small. As this model requires only a very small number of parameters which can be determined to high precision from other data [5], agreement cannot be obtained by adjusting the parameters, leaving open the possibility that exotic components could play a role in the explanation of the effect [6].

In Ref. [7], which we will refer to as GL, we introduced a new relativistic formulation of the impulse approximation (which we refer to as the RIA), based on the relativistic spectator model [8]. Using this formulation, we found that we can explain the EMC effect in the region of large x entirely in terms of conventional nuclear degrees of freedom, as shown in Fig. 1.

In this Letter we reinforce these conclusions by showing that (i) the theoretical uncertainty of the RIA calculation is not large, so that the agreement presented in GL is no accident, (ii) the relativistic effects can be rather simply parameterized, and hence their physical origin easily understood, and (iii) the new relativistic theory removes a descrepancy between the results obtained for the neutron structure function, $F_{2n}(x)$, in two different ways: from measurements of the deuteron, and from global fits to nuclei with all values of A.

To begin the discussion, we recall that the A dependence of the experimental ratio $R_A(x)$ has been generally parametrized as a product of a function of x times a function of A:

$$R_A(x) - 1 \simeq \alpha(x)\beta(A)\,. \tag{1}$$

This is referred to as "factorization". Note that the parameterization [2] $R_A(x) \propto A^{\alpha(x)}$ takes this form if the exponent $\alpha(x)$ is sufficiently small. A factorized form for $R_A(x)$ can also be obtained from a naive nonrelativistic IA description. In this case the structure function, F_2^A, is given by a linear convolution formula [9,10]

$$\left[F_2^A(x)\right]_{IA} = \int_x^\infty dz f_A(z) F_2^N(x/z)\,, \tag{2}$$

where F_2^N is the structure function for an off-shell nucleon (which is assumed to have the same form as the on-shell one), $z = A(k\cdot q)/(P_A\cdot q) \simeq k^+/M_N$ is the light-cone momentum fraction of the struck nucleon with four-momentum $k = (k_o, \mathbf{k})$, M_N and M_A are the masses of the nucleon and nucleus A, respectively, and $f_A(z)$ is the nucleon light cone momentum distribution.

To obtain the factorized form (1) from the convolution formula (2), it is useful to exploit the fact that the nuclear momentum distribution $f_A(z)$ is sharply peaked around $z = 1$, and expand the factor $F_2^N(x/z)$ in Eq. (2) in powers of $(1-z)$ around $z = 1$

$$F_2^N(x/z) = F_2^N(x) + x\frac{\partial F_2^N(x)}{\partial x}(1-z) + \frac{1}{2}\left[x^2\frac{\partial^2 F_2^N(x)}{\partial x^2} + 2x\frac{\partial F_2^N(x)}{\partial x}\right](1-z)^2 + \cdots \tag{3}$$

The coefficients of the expansion, accurate to order $1/M_N$, are therefore

$$\begin{aligned} c_0 &= \int_x^\infty dz f_A(z) \simeq \int_0^\infty dz f_A(z) \simeq A \\ c_1 &= \int_x^\infty dz f_A(z)(1-z) \simeq A\left[\frac{\langle E\rangle}{M_N} - \frac{2}{3}\frac{\langle T\rangle}{M_N}\right] \\ c_2 &= \int_x^\infty dz f_A(z)(1-z)^2 \simeq \frac{2}{3}A\,\frac{\langle T\rangle}{M_N}\,, \end{aligned} \tag{4}$$

where $\langle T\rangle$ is the average kinetic energy of the nucleon in the nucleus and $\langle E\rangle = \langle M_{A-1}\rangle + M_N - M_A$ is the average removal energy, with $\langle M_{A-1}\rangle$ the average mass of the spectator $A-1$ nuclear system (in this discussion we neglect the recoil energy of the $A-1$ system). Details of the derivation of these coefficients (4) are discussed in CL. The coefficient c_0 is just the normalization of the light cone momentum distribution, and c_1 and c_2 can be related to $\langle T\rangle$ and $\langle E\rangle$ by exploiting the connection between $f_A(z)$ and the nucleon three-momentum distribution, $n_A(k)$

$$f_A(z) = 2\pi z\int_0^\infty dk\, k\, n_A(k)\int_{-k}^{k} dk_{\parallel}\,\delta\left(1-z-\frac{\langle E\rangle}{M_N}-\frac{k_{\parallel}}{M_N}\right)$$
$$= 2\pi M_N\, z\int_{k_{min}(z,\langle E\rangle)}^\infty dk\, k\, n_A(k)\,, \tag{5}$$

where $k=|\mathbf{k}|$ is the magnitude of the three momentum of the struck nucleon, and $k_{\parallel}$ its component in the direction of the $\mathbf{q}$.

Note the presence of the the factor z (sometimes referred to as the *flux factor*) in these equations. This quantity was omitted from some early papers on nuclear deep inelastic scattering because incorrect assumptions were made in connecting the relativistic formalism with the nonrelativistic distributions actually used in the calculations. Its effect on nuclear structure functions was emphasized in [10], but here we wish to emphasize that the flux factor does not change the normalization by more than a few percent, which is not numerically significant in the discussion of the EMC effect at large x, and it has no effect on the coefficient c_2. The principal effect of the flux factor is to add the term $-2\langle T\rangle/(3M_N)$ to the c_1 coefficient in Eq. (4). This has a pronounced effect, reducing the size of this coefficient by almost a factor of two, and decreasing the size of the EMC effect predicted by the nonrelativistic impulse approximation (for a detailed discussion, see CL).

A factorized equation of the form (1) can be obtained by using the energy-weighted sum rule [11]

$$\langle E\rangle = \frac{A-2}{A-1}\langle T\rangle + 2\epsilon \approx \frac{3}{2}\langle T\rangle\,, \tag{6}$$

where ϵ is the binding energy per nucleon for nucleus A, and the second expression uses the approximation $2\epsilon\approx\langle T\rangle/2$. With these approximations we obtain

$$\alpha(x) = \frac{3}{2}x\frac{\partial F_2^N(x)}{\partial x} + \frac{1}{3}x^2\frac{\partial^2 F_2^N}{\partial x^2}\,,\qquad \beta(A) = \langle T\rangle/M_N\,. \tag{7}$$

Therefore, in nonrelativistic IA, the A and x dependencies of R_A-1 factor, with the A dependence given entirely by the average kinetic energy of a nucleon in a

nucleus, and the x dependence contained entirely in the term which depends on derivatives of the free structure function F_2^N. In CL it was shown that realistic momentum distributions including NN correlations yield values of $\langle T\rangle$ which are large enough to reproduce the EMC effect at $x<0.5$, but the expansion (7) does not explain the behavior of R_A at higher x (as already shown in Fig. 1).

Calculations performed with the Relativistic Impulse Approximation (RIA) introduce some important corrections to the IA convoution formula, Eq. (2). In RIA, the nuclear structure function becomes

$$\left[F_2^A(x)\right]_{RIA} = \int_x^\infty dz f_A^{RIA}(z,x), \tag{8}$$

with

$$f_A^{RIA}(z,x) = 2\pi M_N\, z\int_{k_{min}(\langle E\rangle_A,z)}^\infty dk\, k\, n_A(k)\widetilde{F}_2^N(y,k)\,, \tag{9}$$

where $y=\eta x/z$ ($\eta = AM_N/M_A\simeq 1$) is the momentum fraction carried by a quark inside the bound nucleon, $\widetilde{F}_2^N(y,k)$ is the structure function for the bound (off-shell) nucleon which depends on the longitudinal and transverse momentum of the nucleon through z and k, and $n_A(k)$ can be related to a covariant nuclear spectral function (which is not known, but is determined by its relation to $n_A(k)$). Further analysis of the relativistic kinematics permits us to express the off-shell nucleon structure function, $\widetilde{F}_2^N(y,k)$, as a product of a relativistic phase space factor, $P(y,y')$ times the on-shell structure function of a shifted argument, $F_2^N(y')$,

$$\widetilde{F}_2^N(y,k) = P(y,y')F_2^N(y') \tag{10}$$

where $P(y,y')$ describs the phase space of the spectator quarks (and satisfies the condition $P(y,y)=1$), and y' is the value of y shifted by the relativistic kinematics:

$$y' = y(1-\tfrac{1}{2}\Delta) + \left\{\sqrt{\left(b^2(y)+\tfrac{1}{2}y\Delta\right)^2 + y(1-y)\Delta} - b(y)\right\}\,, \tag{11a}$$

$$b(y) = \frac{1}{2}\frac{m_X^2/M_N^2}{1-y} - \frac{1}{2}(1-y) \tag{11b}$$

$$\Delta = \frac{m^2-k_\mu^2}{M_N^2} = 1 - \frac{\left(M_A-M_{A-1}\right)^2}{M_N^2} + 2\frac{M_A\left(E_{A-1}-M_{A-1}\right)}{M_N^2}\,, \tag{11c}$$

where $m_X > M_N$ is the mass of the spectator quarks with relativistic phase space $P(y, y')$, m is the mass of the struck quark, $k_\mu^2 = k_0^2 - k^2 \neq m^2$ is the square of the four-momentum of the off-mass-shell struck quark, and M_{A-1} is the mass of the recoiling $A-1$-nuclear system. Note that Δ is a measure of how far the struck quark is off-shell, and is zero if it is on-shell.

This important connection between the off-shell and on-shell nucleon structure function was first obtained in GL. The most significant difference between the non-relativistic convolution formula, Eq. (2), and its relativistic counterpart, Eq. (8), is the explicit dependence of the nucleon structure function, $\widetilde{F}_2^N$, on Δ; when $\Delta = 0$ one can easily see that $y' = y$ and the IA convolution formula, Eq. (2), is recovered. We emphasize that Eqs. (9)–(11c) have been derived from considerations of relativistic kinematics only; any dynamical dependence of $\widetilde{F}_2^N$ on the nuclear medium, i.e. an explicit dependence on k_μ^2 not due to relativistic kinematics, has been disregarded. This means that our relativistic formulae are truly consequences of the assumption that the nucleons are not modified by the nuclear medium.

Both the A and x dependence of the EMC-effect at large x is significantly altered by the parameter Δ, which includes the relativistic corrections to the IA. We can easily display (approximately) the effect of these relativistic corrections by expanding $f_A^{RIA}(z, x)$ in powers of the small quantity Δ. We obtain:

$$f_A^{RIA}(z, x) \simeq f_A(z) F_2^N(y) + \langle\Delta\rangle_\perp G^N(y) + \mathcal{O}(\Delta^2)\,, \tag{12}$$

where

$$G^N(y) = \left.\frac{\partial y'}{\partial \Delta}\right|_{\Delta=0} \times \left.\frac{\partial}{\partial y'}\left[F_2^N(y') P(y, y')\right]\right|_{y=y'}\,, \tag{13}$$

and

$$\langle\Delta\rangle_\perp = 2\pi M_N z \int_{k_{min}(z,\langle E\rangle_A)}^{\infty} dk\, k\, n_A(k)\, \Delta \tag{14}$$

is the average value of Δ over the nucleon transverse momentum k. Note that the first term in Eq. (12) (the one proportional to f_A) is identical to the nonrelativistic IA result given in Eq.(2).

Expanding the first term in Eq. (12) around $z = 1$, as we did before, gives

$$R_A(x) - 1 \simeq \alpha_1(x)\beta_1(A) + \alpha_2(x)\beta_2(A)\,, \tag{15}$$

where $\alpha_1(x)$ and $\beta_1(A)$ are the $\alpha(x)$ and $\beta(A)$ given in Eq. (7), and

$$\begin{aligned} \alpha_2(x) &= \frac{G^N(x)}{F_2^N(x)} \\ \beta_2(A) &= \langle\Delta(z)\rangle = 2\pi M_N \int_0^\infty dz\, z \int_{k_{min}(z,\langle E\rangle_A)}^{\infty} dk\, k\, n_A(k)\, \Delta\,. \end{aligned} \tag{16}$$

The new, relativistic correction term $\beta_2(A)$ is the value of Δ averaged over both longitudinal and transverse momentum variables.

To summarize: the RIA still gives a result in which the x dependence of the nuclear structure function is rescaled by the motion of the nucleons, and the A-dependence is still governed by average properties of nucleon dynamics which can be readily calculated with high accuracy by present day nuclear models. The important difference in the region of large x is that the A and x dependencies of the EMC effect cannot be written as a product of a *single* factor of a function of x times a function of A, but require the sum of two such products. This is a consequence of relativistic effects, which give the second term in Eq. (15).

We now turn to a discussion of the three points mentioned at the beginning of this letter. The shaded area in Fig. 1 are the predictions of the RIA, including *an estimate of the theoretical error*. This error includes uncertainties due to different parametrizations of the free nuclear structure functions (shown by the three dotted lines in the figure) and variations in the nuclear parameters of our theory, namely the average removal energy $\langle E\rangle$, and the value of the mass parameter m_X for the spectator quarks. Of these, the only significant error comes from the dependence of the theory on the (unknown) mass of the spectator quarks (which is expected to be a mass close to, but larger than, M_N). The two solid lines shown in Fig. 1 correspond to $m_X = 940$ and 1800 MeV, showing that the predictions are insensitive to the preceise value of this parameter. We conclude that the agreement between theory and experiment is no accident.

Finally, we turn to the question of the extraction of the neutron structure function from experimental data. The usual way to obtain the neutron structure function is to measure the ratio of the deuteron to proton structure functions. Ignoring nuclear motion effects in the deuteron, this ratio is

$$\frac{F_2^D(x)}{F_{2p}(x)} - 1 = \frac{F_{2n}(x)}{F_{2p}(x)} = R(x)\,. \tag{17}$$

However, if the nucleon structure functions are not modified by the nuclear medium, and if our theory of the EMC effect is good enough, then the ratio $R(x)$ could be, in principle, also extracted from measurements on any other nucleus, even if the nuclear recoil effects are large for that nucleus. The value of $R(x)$ obtained in this way should agree with the result obtained from the deuteron.

by measuring the EMC-effect at large x for a wider number of nuclei.

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